Problem Set 4

This problem set is due Friday, 13 February, at 4 PM. Please **staple** your problem set together and deliver it to John.

Although much of your solutions will be mathematical in nature, your write-up should be as much text as equations/mathematics (perhaps more). Correct answers, poorly justified, will not be worth many points. Things you should explain include: where you found a given equation, which law (eg, "Newton's Second Law") you are using, what mathematical steps you are following, and what assumptions you are making. Also note that your solutions will be easier to follow (and less prone to errors) if you work in analytic equations for as long as possible before you plug in any numbers given in the problem. (This has the added benefit of being generalized so you can plug in new numbers easily if you need to.)

A note on collaboration/help/references: please cite any source (this includes the text, although you can just say, '...Equation 10 in Taylor...') you use as well as acknowledging any help you may have received from the tutors or John. I encourage you to work with buddies, but you must acknowledge their assistance. Furthermore, your write-up must be your own and you must understand everything in it. Failure to acknowledge or cite a source of help is a form of academic dishonesty and will be dealt with accordingly. Also note that looking up solutions (either using previous years' solutions, a previous year's student's solutions, or finding textbook solutions) is strictly forbidden. These problems were designed to aid you in your education. To avoid them is doing yourself a disservice.

Finally, an acknowledgment: many of the problems in this problem set are due in whole or in part to the great Bill Titus of Carleton. Any mistakes are mine, but the good stuff is his.

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0. Feedback (**4 pts**)
   Estimate how long this problem set took you. Also include any questions or comments you have for John about this problem set.
1. Balloon Boy (**23 pts**)
   Remember Falcon Heene, aka, "Balloon Boy"? He was the kid who was believed to be in a balloon drifting across Colorado a few years ago. (Long story short, he wasn't. Long story shorter, the media is easily duped.) But wait, how reasonable was it to believe that there was a kid in there in the first place? The balloon was shaped like a flying saucer (and was seriously a scary spit-and-baling-wire construction), 6.1 m in diameter and about 1.5 m thick.
   a) Estimate the mass of a 6-year-old child. (You may consult growth charts if you haven't sized children lately. Just be sure to cite your sources.) Also estimate the mass of plastic tarp needed to make the balloon and the mass of a sheet of plywood 1 m by 1 m. If it helps, you can assume that the tarp runs about a quarter kilogram per square meter. You can use Google to find the surface mass density of plywood. Assume it's around 1/2" thick.
      What's the total *weight* to be lifted?
   b) Now estimate the volume of helium in the balloon. You can treat the balloon as an ellipsoid (two of the axes are the same size). If helium has a density at standard temperature and pressure of 0.1786 kg/m³, how much weight can the helium balloon lift at sea-level? (*Nota Bene*: I gave you the full widths of the balloon above. Most formulas for the volume of an ellipsoid use the semi-major/minor/whatever-axes. Be careful.)
   c) The atmosphere of a troposphere (the part of the atmosphere that includes us) has a density profile that goes that $\rho(z) = \rho_0 e^{-z/H}$ where ρ_0 is the density of the atmosphere at sea level and H is a

constant called the scale height. It depends on things like the mean molecular mass of the air, g, and the temperature. For normal Earth conditions, $H \approx 8.5$ km.
What's the density of air at a mile altitude, where the incident happened? Use this to find the lifting capacity of the balloon. (You may assume that the density of the helium inside the balloon doesn't change with altitude.)

d) My father, a reporter, has asked you for a short analysis of how likely it was that there was a 6-year-old boy on the balloon. Write a paragraph explaining your analysis of the situation and assessment as to whether or not the kid might have been aboard. This is a slightly tricky writing assignment: you're writing to my father who — and all of his science-geek kids love him anyway — was an English major. On the other hand, when he reports your findings, you want your colleagues to feel like they understand enough of what you did to decide whether they believe you. So you'll have to explain in words what you did above, but go easy on the numbers and no equations. You many want to use either the largest or smallest weight you think plausible that the kid, balloon, and box could have been to make your case more effectively. (In other words, if you want to argue that there certainly could be a kid on there, show that the largest plausible weight was liftable. Or vice versa, as the case may be.)

2. Concept Questions (**14 pts**)
For each of the following questions, just give your answer. No explanation is required. Questions are based on Eric Mazur's *Peer Instruction: A User's Guide*.

a) Imagine holding two identical bricks in place under water. Brick 1 is just beneath the surface of the water, while brick 2 is held about 2 feet down. The force needed to hold brick 2 in place is:

(i) greater
(ii) the same
(iii) smaller

b) Lock and Dam #1 on the Mississippi is a short walk from my old house. One day, while I was watching a barge carrying iron ore in the lock, the barge suddenly dumped its ore over the side into the lock (which was sealed on both ends at the time, so it was a tiny, self-contained body of water). What happened to the water level after the iron was poured in?

(i) It went up
(ii) It stayed the same
(iii) It went down

c) A friend and I have ordered teas at our favorite tea place. She got an iced black tea, I got a hot green tea. Both of our cups are the same size and the liquid is level is identical in both cups, Hers just has ice floating in hers and mine doesn't. (Note that tea is really just water, density-wise. It just tastes much better.) Assuming we forget our teas until they have cooled/warmed to the same temperature (and that the ice is entirely melted), which glass is more full?

(i) My friend's formerly-iced tea
(ii) My formerly-hot tea
(iii) The fluid levels are still the same

d) A stone is launched upward into the air. In addition to the force of gravity, the stone is subject to a frictional force due to air resistance. The time the stone takes to reach the top of its flight path is

(i) larger than
(ii) smaller than
(iii) equal to

the time it takes to return from the top to its original position.

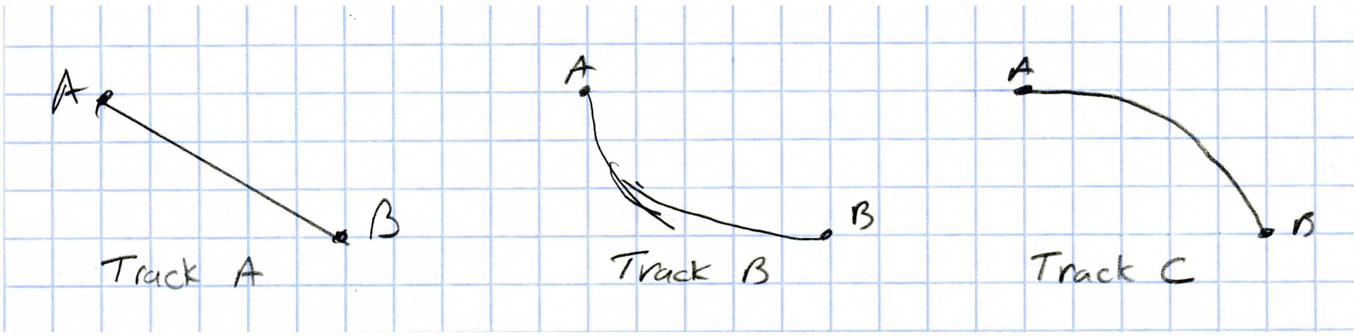

Figure 1: Three tracks that blocks will slide down.

e) Three identical blocks slide without friction along the three tracks shown above, starting at rest at A. Which track will result in the block sliding from A to B in the least time?

(i) Track A
(ii) Track B
(iii) Track C
(iv) They're all the same.

f) Consider the tracks again. Which track will result in the block having the highest speed when it reaches B?

(i) Track A
(ii) Track B
(iii) Track C
(iv) They're all the same.

g) A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed 2v at the bottom, how many times as high must the new ramp be?

(i) $\sqrt{2}$
(ii) 2
(iii) 4
(iv) None of these

3. Particle in a Potential (**21 pts**)
A particle of mass m is moving in a one-dimensional potential, $V(x) = -\frac{cx}{x^2+a^2}$ for positive, real constants a and c and $-\infty < x < \infty$.

a) Find an expression for the force in terms of m, a, c and x.
b) Find an expression for the potential energy in non-dimensional variables.
c) Plot the potential energy diagram in MATLAB.
d) Use the potential energy diagram qualitatively discuss the motion of the particle as a function of it's total energy, E. Be sure to comment on the position as a function of speed! You should find five qualitatively different behaviors depending on the total energy.

4. Euler's Identity (**20 pts**)
Consider Euler's Identity[1]:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \tag{1}$$

This identity is a TYSK. (Know it in your sleep!) It gets used a lot since it relates two seemingly unrelated types of function. Both types of function are vital to all aspects of Physics.
Prove the following identities using Euler's Identity for real θ:

a) $\left|e^{i\theta}\right| = 1$
b) $(e^{i\theta})^* = e^{-i\theta}$ where * indicates a complex conjugate
c) $e^{i\theta_1}e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$
d) $d(e^{i\theta})/d\theta = ie^{i\theta}$
e) $\int e^{i\theta}d\theta = -ie^{i\theta} + C$, where C is a constant.
f) $\sinh(i\theta) = i\sin(\theta)$ for real θ.
g) $\cosh(i\theta) = \cos(\theta)$ for real θ.

[1] One of them, anyway. Prolific bastard has his name on everything, doesn't he? Makes it hard to identify what you mean.