Homework 3

This homework is due Monday, 2 Feb, at 3 PM. Please **staple** your homework together.

Problems marked with a **W** are to be submitted through WebAssign. Problems marked with a **C** are to be turned in in class with complete write-ups. Some problems will be both done on WebAssign and then turned in. For problems that get turned in, start each by starting what general principle(s) you'll be using (eg, "conservation of energy"). If it's a calculation problem, also state clearly what fundamental equation(s) you're going to use. (That is, probably equations you'll find in the equation sheet or in the end of chapter review.)

Although much of your solutions will be mathematical in nature, your write-up should be as much text as equations/mathematics (perhaps more). Correct answers, poorly justified, will not be worth many points. Things you should explain include: where you found a given equation, which law (eg, "Newton's Second Law") you are using, what mathematical steps you are following, and what assumptions you are making. Also note that your solutions will be easier to follow (and less prone to errors) if you work in analytic equations for as long as possible before you plug in any numbers given in the problem. (This has the added benefit of being generalized so you can plug in new numbers easily if you need to.)

A note on collaboration/help/references: please cite any source (this includes the text, although you can just say, '...Equation 10 in Mazur...') you use as well as acknowledging any help you may have received from the tutors or John. I encourage you to work with buddies, but you must acknowledge their assistance. Furthermore, your write-up must be your own and you must understand everything in it. Failure to acknowledge or cite a source of help is a form of academic dishonesty and will be dealt with accordingly. Also note that looking up solutions (either using previous years' solutions, a previous year's student's solutions, or finding textbook solutions) is strictly forbidden. These problems were designed to aid you in your education. To avoid them is doing yourself a disservice.

Many problems will be based on Mazur originals.

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0. Feedback (**C, 2 pts**)
   Estimate how long both the daily and weekly homeworks took you as well as the reading and notes. Provide any feedback you wish to offer.
1. WebAssignCalcPhys1 24.P.001(Problem 1 on WebAssign) (**W, 4 pts**)
   For this problem, we'll use the definition of electric flux, $\Phi = \boldsymbol{E} \cdot \boldsymbol{A}$.
   a) (**2 pts**)
      As we know, the flux is the $\Phi = \boldsymbol{E}\cdot\boldsymbol{A} = |\boldsymbol{E}||\boldsymbol{A}|\cos(90° - \alpha)$, so $\Phi = (1400\,\text{N/C})((4.2\times10^{-3}\text{ m})(4.0\times 10^{-3}\text{ m}))\sin(90 - 42.0°) = 1.57\times10^{-2}\text{ N}\cdot\text{m}^2 C$.
   b) (**2 pts**)
      It would be the same, but in the opposite sense, so that just changes the sign. So we get $\Phi = -1.57\times10^{-2}\text{ N}\cdot\text{m}^2 C$.
2. WebAssignCalcPhys1 24.P.003(Problem 2 on WebAssign) (**W, 6 pts**)
   For the first two parts of this problem, we'll be using geometry and the definition of the area vector (which is perpendicular to the surface). For part c), we'll seen the use the definition of electric flux, $\Phi = \boldsymbol{E}\cdot\boldsymbol{A}$.
   a) (**1 pts**)
      The angle is a right angle with respect to the surface, so it's $90°$ plus the $31.0°$ relative to the x-axis, or $121.0°$.

b) (**1 pts**)
If the flux is zero and the field is not, then the field and the area vectors are perpendicular. Since we know which way the area vector points (right angle to the surface), then the field must point in the plane of the surface. The smallest angle that the field can make and be in the plane of the surface is $31.0°$.

c) (**4 pts**)
The flux is maximum for a given field when it points in the same direction as the area vector so that their dot product is just the product of their magnitudes. The magnitude of the area vector is the area of the surface and the magnitude of the field vector is the magnitude of the field. So we know that $\Phi = 5.04 \times 10^5$ N · m²/C $= EA$. Solving for E, I get

$$\begin{aligned} E &= \Phi/A \\ &= \frac{5.04 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}{\pi(0.159 \text{ m})^2} \\ &= 6.35 \times 10^5 \text{ N/C} \end{aligned} \quad (1)$$

3. WebAssignCalcPhys1 24.P.004(Problem 3 on WebAssign) (**CW, 12 pts**)
I could use the definition of flux for each part and then sum then for the final part, bearing in mind that the area vector always points outwards from a closed surface. But that's the dumb way. The smart way is to use Gauss' law and to remember that in a uniform field, there are no source charge. If there is no source charge, then Gauss' law tells us that the net flux those the surface is zero. (That's the last part done.) Also, all of the faces that are parallel to the field have zero flux since their area vectors are perpendicular to the field. (c, d, and e.) That just leaves a and b. Part a is easy since we can use the definition of flux ($\Phi = \boldsymbol{E} \cdot \boldsymbol{A}$) and the area and field vectors are anti-aligned. The area vector's magnitude is just the area, $(0.164 \text{ m})(0.308 \text{ m}) = 0.0505$ m², and the electric field is given, so the flux is $\Phi = -3460$ N · m²/C. (The minus sign is because the field and area vectors point in *opposite* directions, remember.)
Alright, that just leaves b), the only other non-zero flux...but that has got to be the opposite of part a) since the total has to sum to zero. So it's just 3460 N · m²/C

4. WebAssignCalcPhys1 24.P.007(Problem 4 on WebAssign) (**W, 2 pts**)
For this problem, we'll use the definition of flux again, $\Phi = \boldsymbol{E} \cdot \boldsymbol{A}$. The area vectors point outward from close surfaces, in this case always in the opposite direction of the local electric fields, and have magnitudes equal to the area: $\pi r^2 = \frac{pi/4}{d}^2 = 2.32$ m². So the flux on end 1 is $EA = -89.7$ N · m²/C and 2 is -55.8 N · m²/C. The net flux is then -145 N · m²/C.

5. WebAssignCalcPhys1 24.P.012(Problem 5 on WebAssign) (**CW, 2 pts**)
For this problem, we'll need Gauss's law which states that $\Phi = Q_{\text{enclosed}}/\epsilon_0$. So if $\Phi = 3q/\epsilon_0$, then this implies that $Q_{\text{enc}} = 3q$. We just need to find all of the arrangements that have charges of $3q$ enclosed. That means taking charge q_3 along with one of charges q_1 or q_2, so choices a or g.

6. WebAssignCalcPhys1 24.P.013(Problem 6 on WebAssign) (**W, 6 pts**)
For this problem, we'll use Gauss's law to make a system of equation to solve. Gauss's law states that

$\Phi = Q_{\text{enclosed}}/\epsilon_0$, so we know that

$$\frac{-6q}{\epsilon_0} = \frac{q_1 + q_2}{\epsilon_0} \tag{2}$$
$$\frac{-14q}{\epsilon_0} = \frac{q_1 + q_3}{\epsilon_0} \tag{3}$$
$$\frac{-4q}{\epsilon_0} = \frac{q_2 + q_3}{\epsilon_0} \tag{4}$$

There are a lot of ways to solve this, but I'll solve Eqn (4) for q_3 and then use it in Eqn (3) to solve for q_2. Then I can just solve for q_1. Doing this yields that $q_1 = -8q = -28\ \mu$C. Then it's easy to find that $q_2 = 7\ \mu$C and $q_3 = -21\ \mu$C.

7. WebAssignCalcPhys1 24.P.014(Problem 7 on WebAssign) (**W, 6 pts**)
   For this problem, I'm using Gauss's law,$\Phi = Q_{\text{enclosed}}/\epsilon_0$. We can find $Q_{\text{enclosed}} = q_1 + q_2 + q_3 + q_4 = 22.2\ \mu$C. So the flux is $\Phi = 22.2 \times 10^{-6}$ C$/\epsilon_0 = 2.51 \times 10^6$ N · m²C.
8. Field Line Drawings (**C, 4 pts**)
   For this problem I'm using knowledge of how field line drawings.
   Both are correct. The number of field lines you draw isn't really important, it's the *relative* number that matters. (You should, of course, draw enough lines to give a good, clear sense of the field morphology, but that will depend on each situation and what you think is best. Sorry I can't be more specific.)
9. Sphere to Cube (**C, 4 pts**)
   We'll be using Gauss's law and symmetry here.
   Since the flux through the sphere is the same as the total flux through through the cube. Symmetry tells us that each face of the cube should have the same flux, so each face has one sixth of the total flux, of $\Phi/6$.
   (Note that we cannot get much more specific than this in that we can't say that the flux though various parts of each face is, unlike through the parts of the spherical surface. Symmetry only gets us so far, in this case.)