

Physics 314

Classical and Computational Mechanics

Midterm Exam

This is a midterm exam. You may use a calculator, paper (John can supply you with lots of scratch paper), and a pencil/pen.

You will have 50 minutes to do this exam.

Please write up your solutions on the pad provided. When you are done, staple your work, along with your scratch work at the end.

1. Yo-yo (21 pts)

Consider the humble yo-yo. Say we have a yo-yo made of a disc of mass m and radius R . around the circumference we've wrapped a massless string. The yo-yo is released from rest in $1g$ of gravity. As the yo-yo falls, it unwinds the string. The string is not allowed to slip against the yo-yo.

a) (3 pts)

Draw a diagram of the problem, including a coordinate system.

b) (6 pts)

Find the potential and kinetic energy. (Danger! Danger! There is rotational kinetic energy ($\frac{1}{2}I\omega^2$) in this problem! Don't forget to include it. For a disk of radius R and mass m , $I = \frac{1}{2}mR^2$.)

c) (6 pts)

Find the Lagrangian and then produce the equation of motion for the yo-yo. (Note you'll want to eliminate coordinates so you have just one left.)

d) (6 pts)

Solve the equation of motion.

2. Potential Research (18 pts)

My colleague Arjendu Pattenayak does research on quantum mechanics. In particular, he studies how QM and Classical Mechanics transition from one to the other. His favorite potential is $U(x) = U_0(x^4 - x^2)$ where x is a dimensionless position and U_0 is an energy scale. In his honor, let's take a classical (no quantum required or recommended) look at this potential.

a) (2 pts)

Sketch the potential. (You may want to use some graph paper; if you didn't pick any up, please ask for some.)

b) (2 pts)

Label the equilibrium points and note if each is stable or unstable. Explain why in a sentence.

c) (6 pts)

I see four ranges that the total energy can fall into. ("Ranges" may be true ranges or single, discrete values, here.) CLEARLY label them on your plot. Now explain, in English sentences, what the motion will be in each range.

d) **(8 pts)**

The minimum in the potential occurs as $x_0 = \pm 1/\sqrt{2}$. You may choose either, they're symmetrical, but find the Taylor series in $U(x)$ around that point. (If you wish, you may change variables to $s = x - x_0$.) Go out to the first "interesting" term. Use your Taylor series to find the period of small oscillations about the minimum.