

# Lab 1:

## Eratosthenes and the Size of the Earth

### Purpose

The goal of this activity is to understand both how Eratosthenes measures the size of the Earth and to again grapple with the limitations of his method.

### Directions

We're going to go outside to measure the length of the shadow of a post. Sadly, I don't know anyone living due south of us in the tropics. I do have a friend in Los Angeles who has agreed to measure the length of the shadow, however, and that's even better<sup>1</sup>.

What you'll need:

- A gnomon (a pole whose shadow we will measure)
- A measuring device (either a meter stick or a tape measure)
- A flat, level bit of sidewalk
- Your wits and a calculator

### Before Measuring

Before we go out and measure, let's think about what we'll be measuring and how we'll use it.

Start by copying the figure I have drawn on the front board of the Sun and Earth. Make the Earth big and toward the left and put the Sun waaaaay on the right.

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<sup>1</sup> His name is Kevin Grazier and if you're really, really loyal about reading credits at the end of science fiction movies and shows, you've seen his name.

Why is the angle  $\theta$  between Lacey and LA (measured at the center of the Earth) useful? Well, to start with, I know that the distance (flight, not driving) from Lacey to LA is 1495 km. So what? Well, we can set up a relationship between  $\theta$  and a full circle ( $360^\circ$ ), the Lacey/LA distance ( $D$ ), and the circumference of the Earth ( $C$ ). Set that up now. (Hint: try a few things on some scratch paper. What if  $\theta$  were  $360^\circ$ ?  $180^\circ$ ?  $90^\circ$ ? What's the pattern?) When you have the solution (you can check with me), solve for  $C$ , since that's what we want.

OK, we see that  $\theta$  is important. Unfortunately, we can't measure  $\theta$  directly. What we can measure are  $\alpha$  and  $\beta$ , the angles between the zenith (straight up) and the Sun. Spend some time working out a relationship between these three angles. (Hint: the rays from the Sun are basically parallel, given the distance to the Sun versus the size of Earth.)

I will be circulating to help you out. If you feel stuck, don't panic! Just ask for help! (This is a good place to record your argument. When you think you have it, check with me.)

We're almost there! One last step: measuring  $\alpha$  and  $\beta$ . We won't measure them directly, either. We're going to use some gnomens with height  $h$  that will cast shadows of length  $l$  on the ground. Draw a diagram similar to that on the board showing the gnomon and the shadow. Note that the gnomens will be on level ground, so the angle there is  $90^\circ$ . Now use the definitions of sine, cosine, or tangent to find a relationship between the angle  $\alpha$  or  $\beta$  and  $h$  and  $l$ .

## Measurements and Analysis

Alright, it's measurement time. Once we're all set, we'll all head outside and take our measurements.

	Lacey	LA
h		
l		
$\alpha$ or $\beta$		

Also, the time we measured these was (don't forget the time zone!):

## Analysis

OK, now you can compute  $\theta$ :

And then  $C$  (don't forget units!):

OK, you've got the circumference of the Earth! What radius does that correspond to? How does it compare with the currently accepted<sup>2</sup> equatorial radius of 6378 km? (A percent difference would be ideal to describe it.)

Let's **stretch** things a bit and think about error and uncertainty in general terms. What do you think was your biggest source of error? What assumptions or measurements were probably the least uncertain/inaccurate?

What other problems do you think Eratosthenes might have faced in trying out this measurement that you didn't? (Hint: there are several I can think of, ranging from technology to mathematics to language itself.)

## Write-Up

To finish up, you'll need to write up a short paper describing your findings. Pretend you're submitting this to your friends on campus. Describe your method in enough detail that they'll understand, but not too much so as to bore them. Focus mainly on what you found and how confident you are in your result based on your measurement uncertainties. A few diagrams (hand-drawn are fine) would go a long way to making things clearer, too. Expect to need about 3—5 pages, total, 1.5-spaced.

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<sup>2</sup> Why did I say "currently accepted" rather than "true"?